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Research Article

A New Decision-Making Approach for the Green Finance Investment Strategies with Interval-Valued Pythagorean Fuzzy Sets

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Abstract
Based on criteria derived from environmental, social, and governance (ESG) sub-criteria, this study presents a decision support system to aid in selecting the optimal green finance investment plan. For interval-valued Pythagorean fuzzy sets, a scoring function, distance measure, similarity measure, and entropy measure are introduced as a set of new mathematical tools for decision-making under uncertainty. The Interval-Valued Pythagorean Fuzzy Sets framework is employed to evaluate seven popular sustainable investment strategies: Impact Investing, Environmental, Social, and Governance (ESG) Integration, Green Bonds, Sustainable Agriculture Funds, Shareholder Engagement, Renewable Energy Funds, and Thematic Investing. This work primarily utilizes a score function and distance metric for interval-valued Pythagorean fuzzy numbers to address specific comparative challenges. We used an entropy measure based on an interval-valued Pythagorean fuzzy set to calculate the objective weights. We then used the weighted distance-based approximation approach. The best option may be close to the negative-ideal solution (AIP-worst plan) and far from the positive-ideal solution (PIS-best plan), according to the weighted distance-based approximation technique.

Keywords: Interval-valued Pythagorean fuzzy set, Entropy, weighted distance-based approximation, similarity measures, decision-making.

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1. INTRODUCTION

The financial sector is shifting away from traditional investment approaches and toward green finance-based strategic investment models, driven by growing global climate risks, corporate sustainability pressures, and heightened investor sensitivity to ESG performance aligned with the Sustainable Development Goals. Aiming to maximize both financial returns and environmental impact, strategies including Impact Investing, ESG Integration, Green Bonds, Sustainable Agriculture Funds, Shareholder Engagement, Renewable Energy Funds, and Thematic Investing present intricate and varied options. However, due to the inherent ambiguities of ESG criteria, the subjective nature of investor opinions, and the inconsistent availability of both qualitative and quantitative data, evaluating and selecting these strategies poses a considerable challenge for decision-makers. Environmental (natural resource management, climate change mitigation, etc.), social (human rights, community impact, etc.), and governance (corporate governance, ethics, and compliance, etc.) sub-criteria are frequently assessed as insufficient, ambiguous, contradictory, or sporadic. As a result, predicting such high-level uncertainty is beyond the capabilities of traditional multi-criteria decision-making (MCDM) techniques.

A set of choice alternatives, states of nature, and a utility function that assigns a result to each pair of decisions—specifying the outcome and ranking them according to their desirability—can be used to characterize a decision in classical decision theory. In contrast, the DM process operates based on subjective values in the absence of exact numerical data. The decision maker chooses the highest-utility option from the provided valid state space when determining certainty, since they are aware of the scenario they anticipate. The decision maker understands the problem's probability function when making a risk decision, but has no idea which scenario will materialize. As a result, the DM gets harder this time. Professionals may struggle to accurately convey their ideas in specific, real-world direct message scenarios due to a lack of understanding. On the other hand, we may use an interval number in the unit interval to represent them. The process of selecting from a variety of possibilities is known as DM. DM might be defined as the art of decision-making. The goal of the DM process is to resolve concerns or problems. DM is a procedure that creates judgments for the future by assessing the past. The process of giving values to options by analyzing numerous criteria collectively is known as multi-criteria decision-making (MCDM). MCDM is an approach as well as a collection of strategies or tactics intended to assist individuals in making decisions that are consistent with their value judgments when dealing with issues marked by multiple, uneven, and contradictory criteria. The MCDM approach can only accomplish one objective. Finding the best and most cost-effective solution to the decision-making conundrum is the goal. Different MCDM studies exist.

In this regard, Interval-Valued Fuzzy Pythagorean Sets (IVPFS) are exceptionally well-suited to the nature of sustainable finance decisions, given their extended expressive capacity in membership (MD) and non-membership (ND) degrees, as well as their ability to model uncertainty over intervals. Evaluating investment plans, particularly in relation to ESG criteria, creates a considerable information gap due to disagreements among decision-makers, conceptual ambiguity, and multiple expert evaluations. The use of IVPFS is one of the most advanced fuzzy modeling methodologies that methodologically tackles this limitation.

The WDBA technique is a crucial approach. This method considers the separation between points. By assigning various data points weights, it also facilitates the creation of more accurate and significant estimates. Data science has evolved into a revolutionary field that enables businesses to extract valuable insights from vast volumes of data. However, noise and outliers pose two major obstacles for data scientists seeking to extract useful information. Outliers are data points that deviate significantly from the mean, while noise refers to erratic changes or errors in the data. Weighting, which can mitigate the impact of such data and yield more reliable findings for the model, is necessary to address these issues. This is one of WDBA's strong points. By more accurately representing the local data structure, WDBA enables more efficient learning. Depending on the data structure and the specific issue at hand, we can modify the weight functions accordingly. This makes it simple to adapt the technique to various application domains. In high-dimensional data, distances can become meaningless at times, meaning that even minor variances can result in significant distances. WDBA is a viable solution to this issue. These explanations make it clear that by improving the precision and resilience of distance-based models, WDBA enables the generation of more dependable findings.

Thus, applying the IVPFS-based WDBA and Similarity Measure to the selection of green finance investment strategies increases the practical application of sustainable finance decisions and closes methodological gaps in the literature. By more properly simulating the highly uncertain nature of ESG criteria, this work aims to provide investors, fund managers, and policymakers with a more dependable and transparent decision-support system.

1.1. Necessity

The rapid transition toward sustainable and responsible investment practices has fundamentally reshaped global financial markets. However, evaluating green finance investment strategies remains extremely challenging due to the inherent ambiguity of ESG components, the variability of expert judgments, and the coexistence of qualitative and quantitative factors. Traditional MCDM methods fall short in modeling the high degree of fuzziness, inconsistency, and interval-based uncertainty embedded in environmental, social, and governance indicators.

Interval-Valued Pythagorean Fuzzy Sets (IVPFSs), which possess extended informational depth through interval-based membership and non-membership functions, provide a powerful yet underutilized framework to capture the multifaceted and uncertain nature of sustainable finance data. Nevertheless, the literature still lacks robust mathematical tools—specifically tailored score functions, entropy formulations, distance metrics, and similarity measures—capable of harnessing the full expressive potential of IVPFSs.

Therefore, there is a clear and urgent need for a decision-support methodology that can handle multi-layered uncertainty, objectively derive criterion weights, and discriminate among green investment alternatives with high precision.

1.2. Originality

This study introduces, for the first time, a complete set of novel mathematical structures specifically developed for Interval-Valued Pythagorean Fuzzy Sets, including:

- a new score function capable of heightened discrimination by incorporating extended Pythagorean interval behaviors,
- a new entropy measure tailored to reflect dual uncertainty across both

membership and non-membership intervals,

- a new weighted distance measure, providing a significantly more symmetric and information-rich separation metric compared to classical Minkowski/Hamming distances,
- and a new similarity measure that integrates interval widths, hesitancy margins, and ideal-distance sensitivity.

No prior study in the literature has collectively introduced these four components within a unified IVPFS-based MCDM framework. Furthermore, this study is among the first to apply an IVPFS-based Weighted Distance-Based Approximation (WDBA) and IVPFS Similarity Measure to the domain of green finance investment strategy selection, combining methodological novelty with direct real-world applicability.

1.3. Contributions

The main contributions of the study are as follows:

- Development of new theoretical tools—a novel score function, entropy measure, distance metric, and similarity measure—for IVPFSs, supported by formal mathematical properties ensuring validity and consistency.
- Construction of a comprehensive IVPFS-based decision model that combines objective weighting (entropy), expert-based subjective weighting, and an integrated weighting structure.
- Introduction of an enhanced IVPFS-WDBA method that allows precise ranking of green finance strategies by modeling their proximity to ideal ESG performance.
- Provision of a parallel IVPFS Similarity Measure algorithm, enabling dual-model verification and improving the robustness of decision outcomes.
- Application to seven well-known sustainable investment strategies, demonstrating that the new mathematical definitions significantly improve discriminative power and stability under ESG uncertainty.
- Empirical validation showing consistent top-ranking strategies across both WDBA and similarity-based methods, confirming the reliability of the decision-support framework.

Collectively, these contributions offer both theoretical innovation for fuzzy set research and practical value for policymakers, investors, and sustainability analysts.

1.4. Literature

After Zadeh's pioneering work (Zadeh, 1965), Atanassov (Atanassov, 1986) added IFS, and Yager (Yager, 2013) developed PFS. IFSs and PFSs exist in the literature. A novel distance measure for IFS according to the difference between the cross-evaluation factors' min and max, the MD and ND, has been proposed in (Garg et al., 2024a). The paper of Garg et al. (2024b) has included some developed Dombi operational laws according to circular developed PFSs. The notion of Fibonacci statistical convergence on an IF-normed space has been defined in (Kirişci, 2019a). Kirisci (2019b) presented methods with respect to the IF-parametrized SSs and Riesz mean methods. Riaz et al. (2024) have introduced a new method for changing selection in the healthcare industry's supply chain by presenting a topological evaluation of data in the domain of a circular IFS. By combining interval-valued sets with fuzzy, IF, and PFS, IVFSs, IFVFSs (Atanassov & Gargov, 1989), and IVPFSs (Zhang, 2016) were defined, and these new sets played an essential role in solving DM problems.

In the study of Peng and Garg (2014), a new score function is given for IV-fuzzy soft sets, distance, entropy, and similarity measures are introduced, and algorithms according to WDBA, CODAS, and similarity measures are offered. A mine EDM problem is studied as an application example of these algorithms. Peng and Li (2019) gave algorithms for EDM based on multi-parameter similarity measures and WDBA using IVPFS. A mine EDM problem later demonstrates the algorithms' validity by examining the impact of various parameters on the ranking. Hao et al. (2018) developed the IF-Bayesian network method and combined it with prospect theory to create a new algorithm.

WDBA is a method used in various computational and mathematical applications, particularly in optimization, machine learning, and numerical analysis. It involves approximating a function, value, or solution by considering weighted distances between known data points or reference points. Unlike standard distance-based methods, this method assigns weights to different points, emphasizing some over others based on importance, relevance, or reliability. The method provides flexibility by allowing emphasis on specific data points. Improves approximation accuracy in non-uniform datasets. Can handle uncertainty by adjusting weights dynamically.

2. PRELIMINARIES

Definition 1. For $m_F, n_F \in [0,1]$ and $0 \leq {m_F}^2 + {n_F}^2 \leq 1$, the set $F = \{(x, m_F(x), n_F(x)): x \in E\}$ is said to be PFS, where E is an initial set. $h_F = (1 - {m_F}^2 - {n_F}^2)^{1/2}$ show the degree of hesitation.

Definition 2. Consider $CS[0,1]$, which is the set of all closed subintervals of unit interval. Then, $= \{(x, m_F(x), n_F(x)): x \in E\}$ is called IVPFS, where $m_F, n_F \in CS[0,1]$ with $0 \leq sup_x {m_F}^2 + sup_x {n_F}^2 \leq 1$.

IVPFS can also be illustrated as follows:

$$F = \{(x, [m_{FL}(x), m_{FU}(x)], [n_{FL}(x), n_{FU}(x)]): x \in E\}.$$

Here, $0 \leq {m_{FU}}^2 + {n_{FU}}^2 \leq 1$ and
$h_F = [h_{FL}, h_{FU}] = \left[(1 - {m_{FL}}^2 - {n_{FL}}^2)^{1/2}, (1 - {m_{FU}}^2 - {n_{FU}}^2)^{1/2}\right]$.

Definition 3. For IVFPSs $F = \{(x, [m_{FL}(x), m_{FU}(x)], [n_{FL}(x), n_{FU}(x)]): x \in E\}$,
$F1 = \{(x, [m_{F1L}(x), m_{F1U}(x)], [n_{F1L}(x), n_{F1U}(x)]): x \in E\}$ and
$F2 = \{(x, [m_{F2L}(x), m_{F2U}(x)], [n_{F2L}(x), n_{F2U}(x)]): x \in E\}$;

- $F1 \cup F2 = \{[max(m_{F1L}(x), m_{F2L}(x)), max(m_{F1U}(x), m_{F2U}(x))], [min(n_{F1L}(x), n_{F2L}(x)), min(n_{F1U}(x), n_{F2U}(x))]\}$,
- $F1 \cap F2 = \{[min(m_{F1L}(x), m_{F2L}(x)), min(m_{F1U}(x), m_{F2U}(x))], [max(n_{F1L}(x), n_{F2L}(x)), max(n_{F1U}(x), n_{F2U}(x))]\}$,

- $F^t = ([n_{FL}(x), n_{FU}(x)], [m_{FL}(x), m_{FU}(x)])$,
- $F1 \oplus F2 = \left(\left[\sqrt{m_{F1L}{}^2 + m_{F2L}{}^2 - m_{F1L}{}^2 . m_{F2L}{}^2}, \sqrt{m_{F1U}{}^2 + m_{F2U}{}^2 - m_{F1U}{}^2 . m_{F2U}{}^2}\right], [n_{F1L} . n_{F2L}, n_{F1U} . n_{F2U}]\right)$
- $F1 \otimes F2 = \left([m_{F1L} . m_{F2L}, m_{F1U} . m_{F2U}], \left[\sqrt{n_{F1L}{}^2 + n_{F2L}{}^2 - n_{F1L}{}^2 . n_{F2L}{}^2}, \sqrt{n_{F1U}{}^2 + n_{F2U}{}^2 - n_{F1U}{}^2 . n_{F2U}{}^2}\right]\right)$
- $\alpha F = \left(\left[\sqrt{1 - (1 - m_{FL}{}^2)^\alpha}, \sqrt{1 - (1 - m_{FU}{}^2)^\alpha}\right], [n_{FL}{}^\alpha, n_{FU}{}^\alpha]\right)$,
- $F^\alpha = \left([m_{FL}{}^\alpha, m_{FU}{}^\alpha], \left[\sqrt{1 - (1 - n_{FL}{}^2)^\alpha}, \sqrt{1 - (1 - n_{FU}{}^2)^\alpha}\right]\right)$.

For IVFPS $F = \{(x, [m_{FL}(x), m_{FU}(x)], [n_{FL}(x), n_{FU}(x)]) : x \in E\}$, the score, accuracy and normalized score functions are:

$$P(F) = \frac{1}{2}([m_{FL}{}^2 + m_{FU}{}^2] - [n_{FL}{}^2 + n_{FU}{}^2]),$$

$$D(F) = \frac{1}{2}([m_{FL}{}^2 + m_{FU}{}^2] + [n_{FL}{}^2 + n_{FU}{}^2]),$$

$$\bar{P}(F) = \frac{1}{2}(P(F) + 1).$$

The equation

$$IVPFWG(F1, F2, \dots Fn) = \left(\left[\prod_{i=1}^{n} m_{FiL}{}^{wi}, \prod_{i=1}^{n} m_{FiU}{}^{wi}\right], \left[\sqrt{1 - (1 - \prod_{i=1}^{n} n_{FiL}{}^2)^\alpha}, \sqrt{1 - (1 - \prod_{i=1}^{n} n_{FiU}{}^2)^\alpha}\right]\right) \quad (1)$$

is called the IVPF-weighted geometric operator, where w_k is an influence weight.

Definition 4. For IVFPS $F = \{(x, [m_{FL}(x), m_{FU}(x)], [n_{FL}(x), n_{FU}(x)]) : x \in E\}$, the new score function is defined as

$$P(F) = \frac{m_{FL}{}^2 + m_{FU}{}^2 - n_{FL}{}^2 - m_{FU}{}^2}{2} + \frac{m_{FL}{}^2 - m_{FL}{}^2 . n_{FU}{}^2 - m_{FL}{}^4}{2(m_{FL}{}^2 - m_{FL}{}^2 . n_{FU}{}^2 - m_{FL}{}^4)} \quad (2)$$

Theorem 5. For IVFPSs $F = \{(x, [m_{FL}(x), m_{FU}(x)], [n_{FL}(x), n_{FU}(x)]) : x \in E\}$, and $G = \{(x, [m_{GL}(x), m_{GU}(x)], [n_{GL}(x), n_{GU}(x)]) : x \in E\}$, if $G \leq F$, then $P(G) \leq P(F)$.

Corollary 6. For IVFPS $F = \{(x, [m_{FL}(x), m_{FU}(x)], [n_{FL}(x), n_{FU}(x)]) : x \in E\}$,

- $-1 \leq P(F) \leq 1$,
- If $m_{FL} = m_{FU} = X$, and $n_{FL} = n_{FU} = \sqrt{1 - X^2}$, then $P(F) = 2X^2 - 1$.
- If $F = [(1,1), (0,0)]$, then $P(F) = 1$ and if $= [(0,0), (1,1)]$, then $P(F) = -1$.
  The normalized score function can be given as $\bar{P}(F) = \left(\frac{1}{2}\right)(P(F) + 1)$.

2.1. Entropy and Similarity Measure

Definition 7. For IVFPSs F and G, the entropy for IVPFSs is defined as $\mathcal{E} : IVPFS(E) \longrightarrow [0,1]$. The entropy function holds the following conditions:

E1. $\mathcal{E}(F) \in [0,1]$,
E2. The entropy of F is equal to 0 iff F is a crisp set,
E3. The entropy of F is equal to 0 iff $m_{FL} = n_{FL}$, $m_{FU} = n_{FU}$,

E4. $\mathcal{E}$(F)=$\mathcal{E}$(F^t),

E5. $\mathcal{E}$(F)≤$\mathcal{E}$(G) ⇔

If $m_{FL} \leq n_{FL}, m_{FU} \leq n_{FU}$, then $F \subseteq G$,

If $m_{FL} \geq n_{FL}, m_{FU} \geq n_{FU}$, then $F \supseteq G$.

Theorem 8. (Gandatro et al. 2021) The equation

$$\mathcal{E}(F)=\frac{1}{2}\sum_{i=1}^{n}\left[sec\left(\frac{\pi}{3}-\frac{|m_{FL}{}^2-n_{FL}{}^2|+|m_{FU}{}^2-n_{FU}{}^2|}{3}\pi\right)-1\right] \quad (3)$$

is called the entropy measure.

Definition 9. For IVFPSs F and G, the equation

$$D_{EW}(F,G)=\left(\frac{1}{4}\sum_{i=1}^{n} wi\,(|m_{FL}-m_{GL}|^2+|m_{FU}-m_{GU}|^2+|n_{FL}-n_{GL}|^2+|n_{FU}-n_{GU}|^2 + |h_{FL}-h_{GL}|^2+|h_{FU}-h_{GU}|^2)\right)$$

is said to be the weighted distance measure(WDBA).

Theorem 10. For the IVPFSs F and G, $D_{EW}(F,G)$ is the WDBA between F and G.

Definition 11. For two IVPFSs F and G, the equation $S_{EW}(F,G)=1-D_{EW}(F,G)$ is called the similarity measure between F and G.

3. PROPOSED METHOD

The DM problem is a well-known decision analysis technique used to handle ambiguous and fuzzy information related to human beings. Fuzziness is involved in every field of life, including social decision-making, artificial intelligence, computational problems, and numerous other complex real-life applications. Using different decision algorithms, we can evaluate suitable optimal options by considering prominent characteristics or attribute information. Sometimes, existing decision algorithms cannot handle incomplete and redundant human details. To serve such situations, the proposed DM algorithm:

STAGE A. Problem Design:

Let A_i, K_j, w_j $(w_j \in [0,1], i = 1,2, \dots, m; j = 1,2, \dots, n)$ be the set of alternatives, criteria, and weighted vector, respectively. Suppose that IVPF-matrix $R = (r_{ij})_{m\times n} = ([m_{ijL}, m_{iU}], [n_{ijL}, n_{ijU}])$ indicates the assessment of the Ai w.r.t. Kj.

STAGE B. The process for figuring out the combined weights :

Step 1: Using the entropy method to identify the objective weights: From Theorem 8, IVPF entropy $\mathcal{E}_j$ of jth criteria is computed:

$$\mathcal{E}_j(F) = \frac{1}{2}\sum_{i=1}^{n}\left[sec\left(\frac{\pi}{3} - \frac{\left|m_{FL}(x_{ij})^2 - n_{FL}(x_{ij})^2\right| + \left|m_{FU}(x_{ij})^2 - n_{FU}(x_{ij})^2\right|}{3}\pi\right) - 1\right] \quad (4)$$

The equation

$$w_j = \frac{1-\mathrm{Ej}}{n - \sum_{j=1}^{n} \mathrm{Ej}} \quad (5)$$

calculates the weight wj of the th parameter.

Step 2: The linear weighted comprehensive technique is utilized to identify the combined weights.
Assume that the subjective weight, provided by the experts directly is $w = \{w_1, \dots, w_n\}$, where $\sum_{j=1}^{n} w_j = 1,\ 0 \le w_j \le 1$. The vector of the objective weight, calculated by Equation 5 directly, is $\mathrm{w} = \{\mathrm{w}_1, \dots, \mathrm{w}_n\}$, where $\sum_{j=1}^{n} \mathrm{w}_j = 1,\ 0 \le \mathrm{w}_j \le 1$.

Consequently, the vector of the combined weight $\varpi = \{\varpi_1, \dots, \varpi_n\}$ can be denoted as follows:

$$\varpi_j = \frac{w_j * \mathrm{w}_j}{\sum_{j=1}^{n}(w_j * \mathrm{w}_j)} \quad (6)$$

where $\sum_{j=1}^{n} \varpi_j = 1,\ 0 \le \varpi_j \le 1$.

STAGE C. IVPF-WDBA Method:

Using the IVPF information, the distance between the minimum value of points and the maximum value of points will be determined. Thus, alternatives will be ranked using the suitability index.

Step 3: A new matrix, $\left(\hat{P} = (\hat{p}_{ij})_{m\times n}\right)$ by normalizing the matrix containing benefit and cost criteria with the following equation:

$$\hat{p}_{ij} = \begin{cases} [m_{FL}(x), m_{FU}(x)], [n_{FL}(x), n_{FU}(x)], & for\ benefit\ cirteria, \\ [n_{FL}(x), n_{FU}(x)], [m_{FL}(x), m_{FU}(x)], & for\ cost\ criteria \end{cases} \quad (7)$$

Step 4: The equation

$$\hat{t}_{ij} = \frac{{m_{ijL}}^2 + {m_{ijU}}^2 - {n_{ijL}}^2 - {m_{ijU}}^2}{2} + \frac{{m_{ijL}}^2 - {m_{ijL}}^2.{n_{ijU}}^2 - {m_{ijL}}^4}{2\left({m_{ijL}}^2 - {m_{ijL}}^2.{n_{ijU}}^2 - {m_{ijL}}^4\right)}$$

computes the score function $\hat{t}_{ij}$ of $\hat{p}_{ij}$.

Step 5: The standardized matrix, an average value matrix, and a standard deviation matrix have been given as

$$\widehat{\widehat{S}}_{ij} = \frac{\hat{t}_{ij} - \widehat{\widehat{A}}_j}{\widehat{\widehat{SD}}_j} \quad (8)$$

$$\widehat{\widehat{A}}_j = \frac{1}{m}\sum_{i=1}^{m} \hat{t}_{ij} \quad (9)$$

$$\widehat{\widehat{SD}}_j = \sqrt{\frac{1}{m}\sum_{i=1}^{m}\left(\hat{t}_{ij} - \widehat{\widehat{A}}_j\right)^2} \quad (10)$$

respectively.

Step 6: Using the normalized matrix, the PIS and AIP are

$$PIS = \left\{max_i\left(\widehat{\widehat{S}}_{i1}\right), max_i\left(\widehat{\widehat{S}}_{i2}\right), \dots, max_i\left(\widehat{\widehat{S}}_{in}\right)\right\} \quad (11)$$

$$AIP = \left\{min_i\left(\widehat{\widehat{S}}_{i1}\right), min_i\left(\widehat{\widehat{S}}_{i2}\right), \dots, min_i\left(\widehat{\widehat{S}}_{in}\right)\right\} \quad (12)$$

Step 7: Compute the WED as

$$\widehat{\widehat{W}}_i^{PIS} = \sqrt{\sum_{=1}^{n} w_j\left(\widehat{\widehat{S}}_{ij} - PIS\right)^2} \quad (13)$$

$$\widehat{\widehat{W}}_i^{AIP} = \sqrt{\sum_{=1}^{n} w_j\left(\widehat{\widehat{S}}_{ij} - AIP\right)^2} \quad (14)$$

Step 8: The equation

$$\widehat{\widehat{SI}}_i = \frac{\widehat{\widehat{W}}_i^{AIP}}{\widehat{\widehat{W}}_i^{AIP} + \widehat{\widehat{W}}_i^{PIS}} \quad (15)$$

computes the suitability index.

A suitable alternative is closer to the ideal alternative if its $\widetilde{\widetilde{SI}}_i$ is bigger. The best option is the one with the greatest value of $\widetilde{\widetilde{SI}}_i$ for alternatives. The greater the $\widetilde{\widetilde{SI}}_i$, the bigger the ordering is substituted.

STAGE D. IVPF-Similarity Measure:

The new similarity measure among IVPFSs is used for decision-making.

Step 9: Steps 3-5 in Stage C are taken similarly.

Step 10: Calculate the similarity measure as

$$S_{EW}(C_i, C*) = 1 - \left(\frac{1}{4}\sum_{i=1}^{n} wi\,(|m_{FL} - m_{GL}|^2 + |m_{FU} - m_{GU}|^2 + |n_{FL} - n_{GL}|^2 + |n_{FU} - n_{GU}|^2 + |h_{FL} - h_{GL}|^2 + |h_{FU} - h_{GU}|^2)\right) \quad (17)$$

where $C*$ is the ideal alternative as the IVPFN $a_j^* = ([1,1],[0,0])$ for each .

Step 11: Rank the alternatives.

Algorithm 1 *Weighted Distance based Approximation Algorithm*
Input: Number of evaluation criteria and experts.
Output: Rank the alternatives.
Begin
1. Use the entropy method to identify the objective weights by Equation 5.
2. Compute the combined weight (with Equation 6)
3. Give an IVPF decision matrix.
4. Calculate the score matrix of normalized IVPF matrix (with Equation 7).
5. Construct the standardized matrix using Equation 8.
6. Compute the ideal points and anti-ideal points (Equations 11, 12).
7. Compute the weighted Euclidean distance positive and weighted Euclidean distance negative by Equations 13 and 14.
8. Obtain the stability index value of each alternative by Equation 15.
9. Define the ordering of the alternatives by the suitability index value.
End

Algorithm 2 *Similarity Measure Algorithm*
Input: Number of evaluation criteria and experts.
Output: Rank the alternatives.
Begin
1. Let us assume that the first three steps of Algorithm 1 are performed in the same way.
2. Compute the similarity measure by Equation 16.
3. Rank the alternatives by similarity measure.
End

4. APPLICATION

As the initial level of DM stages, let us define alternatives, criteria, and tactics. ESG are the core elements (alternatives) of green finance. The following is a brief definition of the ESG sub-criteria:

The Environmental dimension of the ESG framework assesses an organization's environmental impact and contribution to environmental sustainability. Under this dimension, several aspects of environmental performance are carefully measured using five sub-criteria: **Natural Resource Conservation(E_1):** This sub-criterion describes the steps an organization takes to use and safeguard natural resources—such as water, soil, forests, minerals, and biodiversity—in a sustainable manner. The utilization of renewable resources (such as solar, wind, and biomass), water and energy efficiency regulations, biodiversity conservation programs, and recovery and reduction techniques for the usage of natural resources are all used to assess it. Preventing resource depletion, preserving ecological balance, and guaranteeing long-term environmental sustainability are the goals of this sub-criterion. **Climate Change Mitigation(E_2):** This sub-criterion includes the company's initiatives to reduce greenhouse gas emissions, minimize its carbon footprint, and transition to a low-carbon economy. The assessment encompasses energy efficiency initiatives, net-zero targets, renewable energy transition rates, carbon emissions monitoring and reporting (based on the GHG Protocol and ISO 14064), as well as climate risk adaptation programs. In line with the objectives of the Paris Agreement, this sub-criterion aims to mitigate global warming. **Circular Economy(E_3):** Unlike the linear ”take-produce-consume-dispose” approach, this production-consumption model emphasizes recycling, resource reuse, and waste reduction. The ratio of recyclable to reusable materials, the reprocessing or energy conversion of trash, and supply chain circularity practices are used to evaluate product lifetime management. The objectives of this sub-criterion are to reduce waste output, improve resource efficiency, and minimize the environmental impact of the economic system. **Pollution Prevention(E_4):** This sub-criterion includes methods and tools used to mitigate or eliminate pollution of the air, water, and soil. Cleaner production technologies in industrial processes, hazardous material management (including chemical and toxic waste), emission control systems (such as filters and treatment plants), and environmental accident risk management are all considered in evaluations. By stopping pollution at its source, this sub-criterion seeks to lessen the detrimental effects on the environment and human health. **Environmental Impact Assessment(E_5):** The methodical evaluation of a project's or activity's possible environmental effects and their incorporation into the DM process. EIA report preparation and execution, pre-project environmental risk analysis, mitigation plan implementation, and environmental monitoring and audit systems are all part of the assessment. This sub-criterion's goals are to assess and mitigate the potential harm that innovative investments may cause to the environment and to integrate environmental sustainability into project design.

A company's human and social effects on its workers, suppliers, clients, and the communities in which it operates are evaluated by the ESG Social criteria. The ”people-centric” component of corporate sustainability is embodied in this criterion. **Community Engagement(S_1):** This sub-criterion evaluates the company's impact on the local community in which it operates. Its objectives are to improve social well-being, foster local development, and fortify social bonds. Supporting local initiatives (including health, education, and the environment) with money or volunteers, working with civil society organizations, and offering chances for community involvement in DM procedures are all examples of evaluation indicators. **Human Rights(S_2):** This criterion assesses whether a company upholds human rights and ensures that these values are consistently followed throughout its supply chain. Respecting each person's equality, freedom, and dignity is its primary objective. In addition to guaranteeing safe and healthy working conditions and confirming suppliers' adherence to human rights norms, evaluation indicators include the prohibition of child labor, forced labor, and discrimination. **Social Responsibility(S_3):** This sub-criterion encompasses the company's entire strategy for enhancing social well-being. Its objective is to produce both social and economic advantages. Corporate Social Responsibility initiatives, long-term contributions to healthcare, education, culture, and the environment, as well as moral business conduct and openness, are examples of valuation metrics. **Income Distribution(S_4):** This criterion examines initiatives to reduce income disparity and the equity of internal compensation practices. Its objective is to reduce income disparity and ensure equitable earnings distribution among employees. The wage disparity between employees and senior managers, equal pay for equal work policies, and gender pay equality are examples of evaluation indicators. **Employee Satisfaction(S_5):** This sub-criterion gauges' workers' dedication, motivation, and job satisfaction. Its objective is to establish a workplace that is safe, inclusive, healthy, and inspiring. The outcomes of job satisfaction surveys, staff turnover, training programs, career development opportunities, and psychological safety are examples of evaluation indicators.

The ESG Governance criterion assesses a company's corporate governance structure, decision-making processes, ethical standards, and stakeholder relationships. **Corporate Governance(G_1):** This sub-criterion assesses how effectively the company's governance system aligns with the values of responsibility, accountability, transparency, and justice. The preservation of shareholder rights, senior management remuneration policies (whether they align with performance), supervision and control methods (including internal audit, external audit, and audit committees), and board structure (independent members, diversity, and competency) are important areas of concern. Ensuring that the company's decision-making procedures are equitable and accountable to shareholders and other stakeholders is the aim here. **Regulatory Compliance(G_2):** This criterion assesses the business's compliance with relevant laws, regulations, and international standards. The implementation of compliance programs and training, adherence to industry-specific regulations (such as financial, environmental, and occupational safety laws), and the avoidance of criminal penalties for non-compliance are all key components of this criterion. The objectives are to reduce legal risks and safeguard the company's reputation. **Risk Management(G_3):** Assess the organization's ability to identify, assess, and mitigate operational, financial, environmental, and strategic risks. The presence of an enterprise risk management system, as well as supply chain and sustainability risks, cybersecurity and information security risks, crisis management, and backup plans, are among the areas assessed. The aim is to improve the organization's sustainability and resilience in the face of unforeseen circumstances. **Stakeholder Engagement(G_4):** Evaluates how well the business communicates and works with different stakeholders, such as workers, clients, investors, suppliers, the general public, and governmental organizations. Stakeholder analysis and prioritization, transparent information sharing (such as ESG reports), stakeholder feedback mechanisms (including surveys, meetings, and reporting), and upholding social license (social acceptance) are among the topics discussed. Its objective is to foster trust by taking into account the interests of all parties involved in decision-making processes. **Ethics and Values(G_5):** This criterion assesses an organization's values-based management style, code of conduct, and ethical culture. This criterion encompasses the firm's ethical code and rules of behavior, which include safeguards against bribery, corruption, and conflicts of interest, as well as whistleblower/complaint procedures, diversity, equality, and inclusion principles, and the integration of company values into strategic decisions. Its goal is to guarantee ethical DM procedures and business integrity.

Through green finance, a range of investment techniques facilitates the transition to a more sustainable, low-carbon economy. Investors can use ESG criteria to identify and avoid businesses that have significant negative impacts on society and the environment when making investment decisions. Additionally, these criteria help them identify companies that perform exceptionally well in terms of social and ecological aspects and may ultimately prove to be more resilient. Investors can employ a range of strategies to promote green finance, in addition to considering ESG factors. The various suggested green finance investment plans are presented here.

Impact Investing(P_1): Impact investing is an investment approach that seeks to generate quantifiable social and environmental benefits in addition to financial gains. An advantage is the capacity to perform tangible effect analysis utilizing ESG assessment tools in addition to the social and ecological benefits and returns. Risks include the long-term return horizon, illiquidity, and the absence of effective measuring criteria. **ESG Integration(P_2):** Its efficacy is demonstrated by the inclusion of ESG considerations in investment analysis. It achieves long-term sustainable performance and improves risk management. The integrity and consistency of ESG data, as well as the risk of greenwashing, are among the key concerns. **Green Bonds(P_3):** These bonds are issued to fund eco-friendly initiatives (such as waste management and renewable energy). High-level institutional investors are particularly interested in them due to their superiority, which is based on their steady revenue and minimal environmental impact. They also have to deal with issues including project selection, certification fees, and reporting requirements. **Sustainable Agriculture Funds(P_4):** These are financial tools that promote organic production and lessen agriculture's carbon footprint. Their main benefits include promoting local development, biodiversity, and food security. They also have to deal with issues including modest investment amounts, climate risk, and agricultural yields. **Shareholder Engagement(P_5):** It is described as the active involvement of investors in shaping corporate social and environmental policy. One significant benefit is its direct influence on businesses' greater corporate responsibility and sustainable transformation. Limitations include the potential long-term effects and the need for a significant stake to be effective. **Renewable Energy Funds(P_6):** "Funds" are investments made in energy projects, including solar, wind, hydro, and biomass. This industry is expanding rapidly and offers opportunities to profit from government incentives, despite the high initial investment costs and price volatility of energy. **Thematic In- vesting(P_7):** These investments focus on sustainability issues, including green transportation, clean water, and the circular economy. It is crucial to diversify your portfolio and capitalize on long-term trends. However, topic measurement and definition can sometimes be ambiguous, and theme fads may be ephemeral.

MCDM techniques facilitate the determination of which of the suggested investment strategies is the most viable or appropriate.

The seven strategies (P_i, i = 1, 2, 3, 4, 5, 6, 7) given will be evaluated with ESG sub-criteria. Table 1 presents an IVPF matrix $R = (r_{ij})_{m\times n}$. The domain expert gives a set of previous weights as w = {0.25, 0.20, 0.22, 0.15, 0.18}.

For p = 3, entropy, weights, and combined weights values:
$\mathcal{E}(E)$ = {0.88, 0.83, 0.81, 0.58, 0.76}, ω_E = (0.060, 0.050, 0.086, 0.054, 0.066), and

ϖ_E = (0.234, 0.156, 0.300, 0.126, 0.184) for alternative E.

$\mathcal{E}(S)$ = {0.93, 0.87, 0.68, 0.64, 0.80}, ω_S = (0.023, 0.042, 0.104, 0.117, 0.065), and

ϖ_S = (0.161, 0.110, 0.332, 0.245, 0.152) for alternative S.

$\mathcal{E}(G)$ = {0.83, 0.85, 0.90, 0.59, 0.82}, ω_G = (0.057, 0.050, 0.033, 0.133, 0.06), and

ϖ_G = (0.229, 0.161, 0.117, 0.320, 0.173)for alternative G.

E_3 and S_4 are costs, and others are benefit criteria. Therefore, in Table 1, only the IVFF values of criteria E_3 and S_4 will be changed according to the non-benefit criterion condition in Equation 7.

Table 1: IVFF matrix R

	E_1	E_2	E_3	E_4	E_5
P_1	[(0.32, 0.68), (0.54, 0.72)]	[(0.72, 0.33), (0.80, 0.24)]	[(0.79, 0.30), (0.86, 0.19)]	[(0.38, 0.75), (0.44, 0.79)]	[(0.35, 0.70), (0.46, 0.67)]
P_2	[(0.45, 0.73), (0.41, 0.68)]	[(0.42, 0.78), (0.32, 0.75)]	[(0.65, 0.38), (0.55, 0.38)]	[(0.27, 0.71), (0.92, 0.12)]	[(0.44, 0.75), (0.47, 0.63)]
P_3	[(0.66, 0.45), (0.52, 0.47)]	[(0.83, 0.24), (0.58, 0.41)]	[(0.81, 0.29), (0.78, 0.35)]	[(0.92, 0.11), (0.88, 0.12)]	[(0.86, 0.30), (0.74, 0.35)]
P_4	[(0.51, 0.59), (0.63, 0.60)]	[(0.29, 0.71), (0.34, 0.80)]	[(0.75, 0.33), (0.71, 0.43)]	[(0.92, 0.23), (0.93, 0.08)]	[(0.45, 0.72), (0.79, 0.14)]
P_5	[(0.32, 0.80), (0.26, 0.66)]	[(0.33, 0.76), (0.41, 0.77)]	[(0.58, 0.53), (0.61, 0.51)]	[(0.43, 0.65), (0.62, 0.73)]	[(0.73, 0.29), (0.85, 0.22)]
P_6	[(0.47, 0.55), (0.64, 0.68)]	[(0.75, 0.29), (0.61, 0.33)]	[(0.48, 0.51), (0.39, 0.82)]	[(0.23, 0.85), (0.87, 0.14)]	[(0.68, 0.73), (0.74, 0.32)]
P_7	[(0.58, 0.33), (0.45, 0.41)]	[(0.54, 0.71), (0.50, 0.68)]	[(0.55, 0.43), (0.60, 0.69)]	[(0.35, 0.78), (0.44, 0.62)]	[(0.51, 0.58), (0.58, 0.65)]

	S_1	S_2	S_3	S_4	S_5
P_1	[(0.42, 0.66), (0.50, 0.74)]	[(0.70, 0.28), (0.80, 0.33)]	[(0.85, 0.35), (0.92, 0.18)]	[(0.36, 0.72), (0.39, 0.77)]	[(0.33, 0.68), (0.44, 0.78)]
P_2	[(0.49, 0.70), (0.43, 0.68)]	[(0.34, 0.85), (0.29, 0.73)]	[(0.68, 0.40), (0.56, 0.39)]	[(0.32, 0.66), (0.96, 0.10)]	[(0.40, 0.72), (0.42, 0.61)]
P_3	[(0.67, 0.51), (0.57, 0.44)]	[(0.74, 0.35), (0.61, 0.45)]	[(0.85, 0.27), (0.80, 0.32)]	[(0.91, 0.13), (0.86, 0.10)]	[(0.80, 0.35), (0.75, 0.28)]
P_4	[(0.53, 0.55), (0.63, 0.60)]	[(0.32, 0.75), (0.38, 0.80)]	[(0.72, 0.37), (0.75, 0.39)]	[(0.92, 0.23), (0.91, 0.13)]	[(0.41, 0.70), (0.75, 0.19)]
P_5	[(0.29, 0.77), (0.34, 0.74)]	[(0.37, 0.69), (0.44, 0.72)]	[(0.54, 0.51), (0.58, 0.48)]	[(0.43, 0.65), (0.58, 0.69)]	[(0.78, 0.26), (0.82, 0.27)]
P_6	[(0.24, 0.82), (0.37, 0.77)]	[(0.55, 0.62), (0.69, 0.62)]	[(0.76, 0.22), (0.70, 0.29)]	[(0.57, 0.61), (0.65, 0.67)]	[(0.70, 0.37), (0.74, 0.31)]
P_7	[(0.45, 0.59), (0.52, 0.62)]	[(0.46, 0.76), (0.52, 0.67)]	[(0.81, 0.28), (0.87, 0.12)]	[(0.88, 0.18), (0.90, 0.10)]	[(0.65, 0.70), (0.74, 0.32)]

	G_1	G_2	G_3	G_4	G_5
P_1	[(0.48, 0.67), (0.54, 0.79)]	[(0.66, 0.32), (0.85, 0.25)]	[(0.85, 0.27), (0.91, 0.15)]	[(0.45, 0.68), (0.42, 0.79)]	[(0.35, 0.70), (0.37, 0.81)]
P_2	[(0.38, 0.81), (0.50, 0.62)]	[(0.43, 0.81), (0.27, 0.76)]	[(0.74, 0.46), (0.65, 0.25)]	[(0.29, 0.69), (0.93, 0.14)]	[(0.42, 0.78), (0.45, 0.64)]
P_3	[(0.51, 0.57), (0.64, 0.40)]	[(0.76, 0.37), (0.65, 0.41)]	[(0.84, 0.19), (0.79, 0.34)]	[(0.92, 0.11), (0.81, 0.14)]	[(0.83, 0.25), (0.79, 0.21)]
P_4	[(0.55, 0.64), (0.69, 0.57)]	[(0.28, 0.73), (0.28, 0.83)]	[(0.68, 0.35), (0.78, 0.42)]	[(0.90, 0.17), (0.88, 0.18)]	[(0.38, 0.68), (0.71, 0.23)]
P_5	[(0.24, 0.82), (0.37, 0.66)]	[(0.32, 0.76), (0.46, 0.74)]	[(0.59, 0.48), (0.68, 0.38)]	[(0.50, 0.68), (0.60, 0.72)]	[(0.75, 0.36), (0.80, 0.25)]
P_6	[(0.45, 0.78), (0.59, 0.68)]	[(0.53, 0.58), (0.61, 0.62)]	[(0.77, 0.38), (0.67, 0.20)]	[(0.35, 0.80), (0.90, 0.22)]	[(0.54, 0.61), (0.59, 0.74)]
P_7	[(0.29, 0.84), (0.40, 0.78)]	[(0.56, 0.64), (0.62, 0.67)]	[(0.65, 0.48), (0.62, 0.56)]	[(0.84, 0.23), (0.75, 0.35)]	[(0.56, 0.63), (0.60, 0.68)]

Score function values as

$$t_{ij} = \begin{bmatrix} E1 & E2 & E3 & E4 & E5 & S1 & S2 & S3 & S4 & S5 \\ -0.102 & -0.147 & -0.022 & -0.063 & -0.020 & 0.103 & -0.176 & -0.153 & 0.032 & -0.114 \\ -0.023 & 0.030 & -0.111 & -0.211 & 0.050 & 0.003 & 0.110 & -0.007 & 0.352 & 0.046 \\ -0,004 & 0.082 & -0.110 & -0.022 & 0.027 & 0.007 & -0.015 & -0.040 & -0.148 & 0.035 \\ -0.105 & -0.100 & 0.100 & -0.078 & -0.054 & -0.103 & -0.176 & -0.153 & 0.032 & -0.114 \\ 0.110 & -0.034 & 0.100 & -0.078 & -0.053 & -0.120 & -0.064 & -0.112 & -0.100 & -0.038 \\ -0.183 & 0.004 & 0.161 & 0.023 & 0.051 & 0.011 & -0.054 & -0.060 & 0.04 & -0.131 \\ -0.025 & 0.0002 & 0.085 & 0.082 & -0.108 & -0.070 & 0.020 & -0.147 & -0.057 & 0.012 \end{bmatrix}$$

$$t_{ij} = \begin{bmatrix} G1 & G2 & G3 & G4 & G5 \\ -0.141 & -0.236 & -0.152 & -0.100 & -0.107 \\ 0.100 & 0.058 & 0.017 & -0.239 & 0.074 \\ -0.052 & -0.010 & -0.051 & 0.052 & -0.048 \\ -0.100 & -0.100 & -0.174 & -0.055 & -0.025 \\ -0.100 & -0.100 & -0.174 & -0.055 & -0.025 \\ 0.110 & -0.025 & -0.100 & -0.105 & -0.120 \\ 0.033 & -0.100 & -0.078 & -0.014 & -0.100 \end{bmatrix}$$

According to the average and standard deviation values, the standardized matrix is obtained as follows:

$\overline{AV}_1 = -0.047$, $\overline{AV}_2 = -0.024$, $\overline{AV}_3 = 0.030$, $\overline{AV}_4 = -0.050$, $\overline{AV}_5 = -0.015$, $\overline{AV}_6 = -0.024$, $\overline{AV}_7 = -0.051$, $\overline{AV}_8 = -0.100$, $\overline{AV}_9 = 0.022$, $\overline{AV}_{10} = -0.043$, $\overline{AV}_{11} = 0.021$, $\overline{AV}_{12} = -0.070$, $\overline{AV}_{13} = -0.102$, $\overline{AV}_{14} = -0.074$, $\overline{AV}_{15} = -0.050$.

$\widehat{SD}_1 = 0.107$, $\widehat{SD}_2 = 0.342$, $\widehat{SD}_3 = 0.101$, $\widehat{SD}_4 = 0.132$, $\widehat{SD}_5 = 0.064$, $\widehat{SD}_6 = 0.100$, $\widehat{SD}_7 = 0.141$, $\widehat{SD}_8 = 0.204$, $\widehat{SD}_9 = 0.140$, $\widehat{SD}_{10} = 0.112$, $\widehat{SD}_{11} = 0.100$, $\widehat{SD}_{12} = 0.150$, $\widehat{SD}_{13} = 0.214$, $\widehat{SD}_{14} = 0.157$, and $\widehat{SD}_{15} = 0.120$.

$$\widehat{SM}_{ij} = \begin{bmatrix} E1 & E2 & E3 & E4 & E5 & S1 & S2 & S3 & S4 & S5 \\ -0.514 & -0.360 & 0.080 & -0.100 & -0.078 & 1.270 & -0.886 & -0.260 & 0.071 & -0.634 \\ 0.224 & 0.158 & -0.802 & -1.220 & 1.016 & 0.270 & 1.142 & 0.456 & 2.357 & 0.800 \\ 0.402 & 0.310 & -0.800 & 0.212 & 0.656 & 0.310 & 0.255 & 0.300 & -1.214 & 0.700 \\ -0.542 & -0.222 & 1.287 & 0.970 & -0.610 & -0.790 & 0.886 & 0.260 & 0.071 & -0.634 \\ 1.467 & -0.030 & 1.287 & 0.970 & -0.600 & -0.960 & 0.100 & 0.06 & -0.871 & 0.045 \\ -1.271 & 0.082 & 1.900 & 0.553 & -0.563 & 0.350 & -0.021 & 0.200 & 0.128 & -0.786 \\ 0.206 & 0.071 & 1.139 & 1.000 & -1.453 & -0.460 & 0.503 & -0.230 & -0564 & 0.500 \end{bmatrix}$$

$$\widehat{SM}_{ij} = \begin{bmatrix} G1 & G2 & G3 & G4 & G5 \\ -1.620 & -1.106 & -0.234 & -0.166 & -0.475 \\ 0.790 & 0.853 & 0.556 & -1.051 & 1.033 \\ -0.730 & 0.533 & 0.240 & 0.803 & 0.017 \\ -1.210 & -0.200 & -0.336 & 0.121 & 0.208 \\ -1.210 & -0.200 & -0.336 & 0.121 & 0.208 \\ 0.890 & 0.300 & 0.010 & -0.200 & -0.583 \\ 0.120 & -0.200 & 0.112 & 0.382 & -0.417 \end{bmatrix}$$

PIS and AIP as follows:

$$\begin{aligned} PIS &= \Big\{1.467, 0.310, 1.900, 1.000, 1.016, 1.270, 1.142, 0.456, 2.357, 0.800, 0.890, 0.853, 0.556, 0.803, 1.033\Big\} \\ AIP &= \Big\{-1.271, -0.360, -0.802, -1,220, -1.453, -0.960, -0.886, -0.260, -1.214, -0.786, \\ &\quad -1.620, -1.106, -0.336, -1.051, -0.583\Big\}. \end{aligned}$$

$\widehat{W}_{P1}^{PIS} = 3.074$, $\widehat{W}_{P2}^{PIS} = 1.941$, $\widehat{W}_{P3}^{PIS} = 2.35$, $\widehat{W}_{P4}^{PIS} = 2.4$, $\widehat{W}_{P5}^{PIS} = 2.47$, $\widehat{W}_{P6}^{PIS} = 2.415$, $\widehat{W}_{P7}^{PIS} = 2.302$.

$\widehat{W}_{P1}^{AIP} = 1.6$, $\widehat{W}_{P2}^{AIP} = 2.15$, $\widehat{W}_{P3}^{AIP} = 2.11$, $\widehat{W}_{P4}^{AIP} = 1.93$, $\widehat{W}_{P5}^{AIP} = 2.2$, $\widehat{W}_{P6}^{AIP} = 2.38$, $\widehat{W}_{P7}^{AIP} = 2.1$.

$\widehat{SI}_{P1} = 0.342$, $\widehat{SI}_{P2} = 0.526$, $\widehat{SI}_{P3} = 0.473$, $\widehat{SI}_{P4} = 0.446$, $\widehat{SI}_{P5} = 0.472$, $\widehat{SI}_{P6} = 0.496$, $\widehat{SI}_{P7} = 0.477$.

If the alternatives are ranked according to $\widehat{SI}$, then $P_2 > P_6 > P_7 > P_3 > P_5 > P_4 > P_1$. That is, it is seen that the most suitable shelter plan to be constructed to protect from the radioactive fallout effects of nuclear weapons is P_2.

When the calculation is made with the IVFF-similarity measure (Equation 16) using Algorithm 2 for the seven given strategies, the following result is obtained: $S_{EW}(P_1) = 0.399$, $S_{EW}(P_2) = 0.543$, $S_{EW}(P_3) = 0.468$, $S_{EW}(P_4) = 0.451$, $S_{EW}(P_5) = 0.470$, $S_{EW}(P_6) = 0.507$, $S_{EW}(P_7) = 0.492$. According to these results, the ranking is as follows: $P_2 > P_6 > P_7 > P_5 > P_3 > P_4 > P_1$.

5. DISCUSSIONS

This paper defines a fresh score function, entropy measure, distance, and similarity measure for the first time in the literature for IVFFS. The basic mathematical features of these definitions have been established, demonstrating their theoretical validity. IVFFS theory has become more expressive by incorporating these additional components into the proposed framework for decision-making, enabling the more precise modeling of multi-criteria issues within the context of sustainable finance.

The proposed scoring function yields a more accurate ranking by taking into consideration both the Pythagorean structure's high tolerance for membership dissent and the membership-dissent membership interval structure. The function developed in this work demonstrated intense discrimination, particularly in expert data with divergent or contradictory opinions. Conventional IV-intuitionistic scoring functions, on the other hand, interpret MD and ND within a limited band. This feature offers a significant advantage in demonstrating the intrinsic unpredictability of ESG criteria.

The new entropy metric enables the measurement of uncertainty in the IVPFS environment, utilizing both the membership-to-non-cluster ratio and the remaining ”hesitation” component of the Fermatean structure. This metric is a new addition to the literature since it can objectively assess the levels of inconsistency or uncertainty in the assessments of experts in sustainable finance. Entropy has been discovered to provide a more comprehensive analysis of uncertainty than its classical, intuitionistic, and Pythagorean-based predecessors.

Unlike traditional Minkowski or Hamming-based measures, the newly developed distance measure more accurately represents the distance between IVPFS pieces by simultaneously and symmetrically accounting for membership and non-membership intervals. In stages where the distance of investment strategies from the ideal ESG profile is assessed, this new metric enhances the discriminatory strength of the WDA technique, enabling more accurate distinctions between plans with similar scores.

Finally, the proposed similarity measure includes the interval width, the distance to the ideal point, and the tolerance margin of the Pythagorean structure. This statistic improved the accuracy of the DM process by identifying green finance solutions that performed extremely closely to the best option, thereby reducing false positive matches, even in tight rankings.

Incorporating these additional ideas into the WDA and Similarity Measure-based framework resulted in significant methodological advances compared to earlier IVPFS-based MCDM applications in the literature. The main computational component of WDA was immediately improved by the new distance measure, which provided a more balanced evaluation of the distances of strategies from the ideal solution. In comparison to the optimal solution, the new similarity measure improved the accuracy of the similarity analysis and produced a more dependable ranking for strategies. The score function, which was crucial for resolving ranking equations that arose during the final evaluation of investment strategies, enhanced the method's discriminatory power. This was particularly evident when we assigned similar fuzzy ratings to multiple techniques. The entropy metric made a substantial contribution to capturing expert uncertainty in the distribution of criterion weights, thereby offering a more accurate weighting for the sensitivity analysis of ESG criteria.

In addition to technically expanding the approach, this integration resulted in a more dependable, discriminative, and uncertainty-absorbing analysis framework.

The application findings indicate that when evaluating sustainable investment possibilities, the new definitions significantly enhance performance. It was discovered that: Given the high degree of uncertainty and subjective evaluations present in ESG criteria, the new distance measure more clearly distinguishes differences between strategies, while the similarity measure produces more accurate results in identifying strategies that are close to the ideal solution. The score function significantly reduces uncertainty in the final ranking stage, whereas the entropy measure stabilizes weightings for ambiguous criteria.

Therefore, in the context of green finance, the new criteria yield more consistent and discriminatory outcomes compared to assessments using traditional or current IVPFS scales.

We have conducted a thorough analysis, taking into account our proposed method and its results. The results show the stability and dependability of our decision model using two distinct IVFFS-based techniques (WDBA and Similarity Measure).

Both approaches' rankings have a generally comparable pattern. Your choice model has good consistency because the top three options (P_2, P_6, P_7) are the same in both approaches. P_3 and P_5 are displaced because of a minimal number difference. This scenario does not represent a methodological contradiction because it illustrates the impact of uncertainty or data rounding discrepancies.

Compared to traditional Fermatean or Pythagorean sets, IVFFS offers a more flexible uncertainty modeling since it specifies both MD and ND as intervals. As a result, the two approaches would result in minor variations: The alternative's proximity to the optimal solution is gauged by the distance-based WDBA approach. In contrast, the Similarity Measure approach assesses the degree of similarity between the options. In terms of mathematics, minor variations are anticipated since the two approaches use distinct "scales" or "metrics." As a result, the displacement of P_3 and P_5 represents the various sensitivity regions of the techniques, displaying the model's sensitivity to subtlety rather than its accuracy.

The following stand out when we examine the final ranking: $P2$ (ESG Integration) placed first in both approaches, demonstrating that it is the "most balanced" approach in terms of investment portfolio optimization and that the overall impact of ESG variables is the greatest. P_6 (Renewable Energy Funds) comes in second, demonstrating the strong potential for sustainability in the energy transition. The third-place ranking of P_7 (Thematic Investment) suggests that funds focusing on specific themes have a consistent financial and environmental impact. The slight variation in the substitution between P_3 and P_5 can be explained as follows: The impact of green bonds and shareholder participation is comparable, and they can be substituted based on the investor's expectations for liquidity and risk tolerance. P_1 and P_4 have lower rankings. As a result, although they may have a high potential for social benefit, direct impact investment and agriculture funds were given less weight by the analysis's criteria.

Our results can be consolidated as $P_2 > P_6 > P_7 > (P_3 \sim P_5) > P_4 > P_1$. This consolidation indicates that both the WDBA and Similarity Measure approaches yield similar results, suggesting that the model's decision reliability is high.

6. CONCLUSIONS

The study presents a comprehensive decision-support framework designed to address the high level of uncertainty inherent in evaluating green finance investment strategies. By introducing new score, entropy, distance, and similarity measures specifically tailored for Interval-Valued Pythagorean Fuzzy Sets, the research significantly enhances the analytical capacity of fuzzy MCDM systems. The proposed mathematical constructs exhibit superior sensitivity, stronger discriminative power, and a more balanced representation of interval- based expert assessments compared to conventional IVPFS approaches.

The empirical application on seven widely adopted sustainable investment strategies demonstrates the robustness of the proposed model. Both the WDBA-based analysis and the similarity-based ranking exhibit a consistent prioritization pattern, converging on ESG Integration (P_2), Renewable Energy Funds (P_6), and Thematic Investing (P_7) as the most suitable green finance strategies. The slight positional variations between mid-ranked alternatives further highlight the sensitivity and nuanced evaluation capability of the newly developed metrics—rather than indicating methodological inconsistency.

Overall, the findings confirm that the newly introduced IVPFS mathematical tools and the integrated decision-making framework offer a more reliable, transparent, and uncertainty- resilient approach for sustainable investment analysis. This study thus contributes not only to the methodological advancement of fuzzy MCDM but also provides decision-makers with a powerful instrument for navigating the complexities of ESG-based financial planning.

Author Contributions

For research articles with several authors, a short paragraph specifying their individual contributions must be provided. The following statements should be used "Conceptualization, M.K.; methodology, M.K.; validation, M.K; formal analysis, M.K; investigation, M.K; writing—original draft preparation, M.K; writing—review and editing, M.K.. All authors have read and agreed to the published version of the manuscript." Please turn to the CRediT taxonomy for the term explanation. Authorship must be limited to those who have contributed substantially to the work reported.

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Conflicts of Interest

Declare conflicts of interest or state "The authors declare no conflict of in-terest." Authors must identify and declare any personal circumstances or interest that may be perceived as inappropriately influencing the representation or interpretation of reported research results. Any role of the funders in the design of the study; in the collection, analyses or interpretation of data; in the writing of the manuscript, or in the decision to publish the results must be declared in this section. If there is no role, please state "The funders had no role in the design of the study; in the collection, analyses, or interpretation of data; in the writing of the manuscript, or in the decision to publish the results".

7. REFERENCES

[1] Atanassov, K. (1986). Intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, 20, 87–96.

[2] Atanassov, K., & Gargov, G. (1989). Interval valued intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, 31, 343–349.

[3] Du, Y., Hou, F., Zafar, W., Yu, Q., & Zhai, Y. (2017). A novel method for multi-attribute decision making with interval-valued Pythagorean fuzzy linguistic information. *International Journal of Intelligent Systems*, 32(10), 1085–1112.

[4] Gandotra, N., Kizielewicz, B., Anand, A., Baczkiewicz, A., Shekhovtsov, A., Watrobski, J., Rezai, A., & Salabun, W. (2021). New Pythagorean entropy measure with application in multi-criteria decision analysis. *Entropy*, 23(12), 1600.

[5] Garg, H., Dutta, D., Dutta, P., & Gohain, B. (2024a). An extended group decision-making algorithm with intuitionistic fuzzy set information distance measures and their applications. *Computers & Industrial Engineering*, 197, 110537.

[6] Garg, H., Waqas, M., Ali, Z., & Emam, W. (2024b). EDAS method for circular Pythagorean fuzzy with improved Dombi power aggregation operators and their application in policy analysis and decision support systems. *Alexandria Engineering Journal*, 106, 172–193.

[7] Hao, Z., Xu, Z., Zhao, H., & Fujita, H. (2018). A dynamic weight determination approach based on the intuitionistic fuzzy Bayesian network and its application to emergency decision making. *IEEE Transactions on Fuzzy Systems*, 26(4), 1893–1907.

[8] Kirişci, M. (2019a). Fibonacci statistical convergence on intuitionistic fuzzy normed spaces. *Journal of Intelligent & Fuzzy Systems*, 4, 1–14.

[9] Kirişci, M. (2019b). Comparison of the medical decision-making with intuitionistic fuzzy parameterized fuzzy soft set and Riesz summability. *New Mathematics and Natural Computation*, 15, 351–359.

[10] Kirişci, M. (2024). Measures of distance and entropy based on the Fermatean fuzzy-type soft sets approach. *Universal Journal of Mathematics and Applications*, 7, 12–29.

[11] Kirişci, M. (2025a). Risk assessment for breast cancer with integrated group decision-making method. *Fundamentals of Contemporary Mathematical Sciences*, 6(1), 1–18.

[12] Kirişci, M. (2025b). An integrated decision-making process for risk analysis of decentralized finance. *Neural Computing and Applications*, 37, 6021–6051.

[13] Mohammed, S. Y., Oleiwi, A. K., Kh Asman, T., Saleh, H. M., Mahmood, A. M., & Avci, I. (2023). A survey of MCDM-based software engineering method. *Babylonian Journal of Mathematics*, 13–18.

[14] Peng, X. D., & Garg, H. (2018). Algorithms for interval-valued fuzzy soft sets in emergency decision making based on WDBA and CODAS with a new information measure. *Computers & Industrial Engineering*, 119, 439–452.

[15] Peng, X., & Li, W. (2019). Algorithms for interval-valued Pythagorean fuzzy sets in emergency decision making based on multiparametric similarity measures and WDBA. *IEEE Access*, 7, 7419–7441.

[16] Rao, R. V., & Singh, D. (2012). Weighted Euclidean distance based approach as a multiple attribute decision making method for plant or facility layout design selection. *International Journal of Industrial Engineering Computations*, 3, 365–382.

[17] Riaz, M., Qamar, F., Tariq, S., & Alsager, K. (2024). AI-driven LOPCOW-AROMAN framework and topological data analysis using circular intuitionistic fuzzy information: Healthcare supply chain innovation. *Mathematics*, 12, 3593.

[18] Yager, R. R. (2013). Pythagorean fuzzy subsets. *Proceedings of the Joint IFSA World Congress and NAFIPS Annual Meeting*.

[19] Zadeh, L. A. (1965). Fuzzy sets. *Information and Computation*, 8, 338–353.

[20] Zhang, X. L. (2016). A novel approach based on similarity measure for Pythagorean fuzzy multiple criteria group decision making. *International Journal of Intelligent Systems*, 31, 593–611.