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Research Article

Approximate Solutions for Some Nonlinear Second-Order Ordinary Differential Equations Using Picard–Lindelof and Adomian Decomposition Methods

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Abstract
This paper introduces a comparison between the Picard–Lindelof method (PIM) and the Adomian decomposition method (ADM) for solving second-order nonlinear differential equations. Both methods are applied to selected test problems to evaluate their accuracy and convergence rate. The Picard method is employed as an iterative scheme for successive approximations, while ADM provides a decomposition-based approach to handle nonlinearities. Numerical results indicate that both methods are effective, yet their performance varies depending on the form and complexity of the nonlinear terms. Through Examples 1–4, the paper presents a comparison between the two methods with respect to the exact solution.

Keywords: Picard method, Adomian decomposition method, nonlinear differential equation.

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1 INTRODUCTION

Second-order nonlinear differential equations play a fundamental role in modeling complex physical, biological, and engineering systems. Their analytical treatment is often challenging due to the interaction between nonlinear terms and variable coefficients. In particular, the presence of singular structures within the domain may significantly influence the convergence behavior of approximation techniques [1, 2]. Iterative procedures remain among the most reliable approaches for handling such problems. The classical Picard iteration method is based on successive approximations and provides a constructive framework for analyzing existence, uniqueness, and convergence of solutions [3–5]. Despite its conceptual simplicity, its rate of convergence may vary depending on the structure of the nonlinear components. An alternative analytical strategy is the (ADM), which constructs the solution in a rapidly convergent series without requiring linearization or perturbation assumptions [6–8]. The method has been successfully applied to a wide class of nonlinear ordinary differential equations, including problems with singular coefficients [9, 10]. Although both methods are grounded in iterative concepts, the differences may arise in their computational efficiency and convergence characteristics. Previous comparative investigations have reported close relationships between the two approaches in certain classes of problems. In this work, we examine second-order nonlinear differential equations under both regular and singular settings. A systematic comparison between the (PIM) and the (ADM) is presented through selected test problems, and the numerical results are evaluated against exact solutions whenever available

$$y^{''} + p(x)y^{'} = h(x) + q(x, y), \tag{1.1}$$
$$y(0) = c_1, y^{'}(0) = c_2$$

Where $q(x, y)$ is nonlinear function and $p(x), h(x)$ are given functions and A, B , are constants. In recent years a large amount of research developed concerning picard method , and the related modification to investing various scientific models. Large classes of linear and nonlinear differential equations can be solved using the picard iteration method and the Adomian decomposition method. The (PIM) was first compared with (ADM) by Rach and Sarafyan on a number of examples. Golberg showed that the (ADM) approach to linear differential equations was equivalent to the classical method of successive approximations (PIM) [11–13].

2 ANALYSIS OF THE METHOD

In the process of solving nonlinear second-order ordinary differential equation, we focus on equation of the form given in (1.1), we get

$$Ly = h(x) + f(x, y). \tag{2.1}$$

Where the differential operator L is defined by

$$L(\cdot) = e^{-\int p(x)dx} \frac{d}{dx} e^{\int p(x)dx} \frac{d}{dx}(\cdot). \tag{2.2}$$

The L^{-1} is

$$L^{-1}(\cdot) = \int_0^x e^{-\int p(x)dx} \int_0^x e^{\int p(x)dx}(y^{''} + p(x)y^{'})dxdx. \tag{2.3}$$

Applying L^{-1} of (2.3) to the two terms of (1.1) we obtain

$$y = \omega(x) + L^{-1}(h(x)) + L^{-1}f(x, y). \tag{2.4}$$

Where $L\omega(x) = 0$, from (2.4) we obtain

$$y_0 = \omega(x) + L^{-1}(h(x)) \tag{2.5}$$

in particular, we define

$$y_{n+1} = y_0 + \int_0^x e^{-\int p(x)dx} \int_0^x e^{\int p(x)dx} f(x, y_n)dxdx, \quad n \geq 0 \tag{2.6}$$

2.1 The ADM

The (ADM) usually formulates the equations in an operator form by considering the highest order derivative in the problem. We can write Eq. (1.1) as follows:

$$Ly = h(x) + q(x, y), \tag{2.7}$$

where

$$L(\cdot) = e^{-\int p(x)dx} \frac{d}{dx} e^{\int p(x)dx} \frac{d}{dx}(\cdot), \tag{2.8}$$

we obtain Eq. (2.3).

By applying L^{-1} on (1.1), we have

$$y = \omega(x) + L^{-1}h(x) + L^{-1}q(x, y), \tag{2.9}$$

where ω represents the constants of integration, determined by the initial or boundary conditions depending on whether the problem is an initial-value problem or a boundary-value problem [11–13]. In ADM, the solution y is expressed as an infinite series:

$$y(x) = \sum_{n=0}^{\infty} y_n(x). \tag{2.10}$$

The nonlinear term $q(x, y)$ is then represented by an infinite series as:

$$q(x, y) = \sum_{n=0}^{\infty} A_n, \tag{2.11}$$

Substituting Eq. (2.11) and Eq. (2.12) into Eq. (2.10) becomes

$$\sum_{n=0}^{\infty} y_n = \omega(x) + L^{-1}h(x) + L^{-1} \sum_{n=0}^{\infty} A_n. \tag{2.12}$$

This leads to the recursive relationship for the solution:

$$\begin{aligned} y_0 &= \omega(x) + L^{-1}h(x), \\ y_{n+1} &= L^{-1} \sum_{n=0}^{\infty} A_n, \quad n \geq 0. \end{aligned} \tag{2.13}$$

From Eq. (2.13), we obtain:

$$\begin{aligned}
y_0 &= \omega + L^{-1}h(x),\\
y_1 &= L^{-1}A_0,\\
y_2 &= L^{-1}A_1,\\
y_3 &= L^{-1}A_2,\\
&\vdots
\end{aligned}$$

Where A_n's represent the Adomian polynomials. The first few Adomian polynomials are given by:

$$\begin{aligned}
A_0 &= F(y_0),\\
A_1 &= y_1F'(y_0),\\
A_2 &= y_2F'(y_0) + \frac{1}{2!}y_1^2F''(y_0),\\
A_3 &= y_3F'(y_0) + y_1y_2F''(y_0) + \frac{1}{3!}y_1^3F'''(y_0),
\end{aligned} \tag{2.14}$$

3 APPLICATIONS

In this section we will present some examples through which we compare the methods of Picard and Adomian along with the exact solution.

Example 3.1
Consider the following nonlinear second-order ordinary differential equation:

$$y'' + \frac{7}{x}y' = 4xe^{x^2} + 16e^{x^2} - e^{2x^2} + y^2. \tag{3.1}$$

With initial conditions:

$$y(0) = 1, \quad y^{'}(0) = 0.$$

The exact solution is:

$$y(x) = e^{x^2}.$$

Solution using PIM

The differential operator of (3.1) is

$$L(\cdot) = x^{-7}\frac{d}{dx}x^7\frac{d}{dx}(\cdot),$$

and the inverse operator is

$$L^{-1}(\cdot) = \int_0^x x^{-7}\int_0^x x^7(\cdot)dx.$$

Eq. (3.1) can be rewritten in a simplified form:

$$Ly = 4xe^{x^2} + 16e^{x^2} - e^{2x^2} + y^2. \tag{3.2}$$

Applying the operator to obtain the solution components:

$$y = 1 + \frac{15x^2}{16} + \frac{4x^3}{27} + \frac{7x^4}{20} + L^{-1}y_n^2. \tag{3.3}$$

Using the Picard Method, we define the following iteration scheme:

$$y_0 = 1 + \frac{15x^2}{16} + \frac{4x^3}{27} + \frac{7x^4}{20}, \quad y_{n+1} = y_0 + L^{-1}y_n^2.$$

Further Expressions:

$$y_1 = 1 + x^2 + \frac{4x^3}{27} + \frac{127x^4}{320}.$$

$$y_2 = 1 + x^2 + \frac{4x^3}{27} + \frac{2x^4}{5}$$

Solution using ADM

We now solve Equation (3.1) using the Adomian decomposition method. We can rewrite (3.1) as:

$$Ly = 4xe^{x^2} + 16e^{x^2} - e^{2x^2} + y^2. \tag{3.4}$$

The corresponding inverse operator is given by

$$L^{-1}(\cdot) = x^{-7}\frac{d}{dx}\left(x^7\frac{d}{dx}(\cdot)\right).$$

Applying L^{-1} to (3.4) and using the initial conditions, we get

$$y(x) = 1 + \frac{15x^2}{16} + \frac{4x^3}{27} + \frac{7x^4}{20} + L^{-1}\left(4xe^{x^2} + 16e^{x^2} - e^{2x^2} + y^2\right). \tag{3.5}$$

By assuming that:

$$y(x) = \sum_{n=0}^{\infty} y_n(x).$$

By substituting in the equations, we get:

$$\sum_{n=0}^{\infty} y_n(x) = 1 + \frac{15x^2}{16} + \frac{4x^3}{27} + \frac{7x^4}{20} + L^{-1}\left(\sum_{n=0}^{\infty} A_n\right).$$

Where

$$y_0 = 1 + \frac{15x^2}{16} + \frac{4x^3}{27} + \frac{7x^4}{20}, \quad y_{n+1} = L^{-1}\left(\sum_{n=0}^{\infty} A_n\right),$$

Where the Adomian polynomials are given by:

$$A_n = y_n^2.$$

The first few components of the Adomian polynomials are:

$$A_0 = y_0^2.$$

$$A_1 = 2y_0y_1.$$

$$A_2 = 2y_0y_2 + y_1^2.$$

Hence:

$$y_0 = 1 + \frac{15x^2}{16} + \frac{4x^3}{27} + \frac{7x^4}{20}$$
$$y_1 = \frac{x^2}{16} + \frac{3x^4}{64}$$
$$y_2 = \mathcal{O}(x^{14})$$

So,

$$y(x) = y_0 + y_1 + y_2 = 1 + x^2 + \frac{4x^3}{27} + \frac{127x^4}{320} + \mathcal{O}(x^{12})$$

Table 1: The comparison between exact solution $y(x) = e^{x^2}$ and PIM, ADM.

x	Exact	PIM	ADM	Exact-PIM	Exact-ADM	PIM-ADM
0.0	1.00000	1.00000	1.00000	0.0000	0.0000	0.0000
0.1	1.01005	1.01019	1.01019	0.000137981	0.000137669	0.0000003125
0.2	1.04081	1.04183	1.04182	0.00101441	0.00100941	0.000005
0.3	1.09417	1.09724	1.09721	0.00306572	0.0030404	0.0000253125
0.4	1.17351	1.17972	1.17964	0.00621061	0.00613061	0.00008
0.5	1.28403	1.29352	1.29332	0.0094931	0.00929779	0.000195313
0.6	1.43333	1.44384	1.44344	0.0105106	0.0101056	0.000405
0.7	1.63232	1.63685	1.6361	0.00453859	0.00378828	0.000750313
0.8	1.89648	1.87969	1.87841	0.016789	0.018069	0.00128
0.9	2.24791	2.18044	2.17839	0.067468	0.0695183	0.00205031
1.0	2.71828	2.54815	2.54502	0.170134	0.173259	0.003125

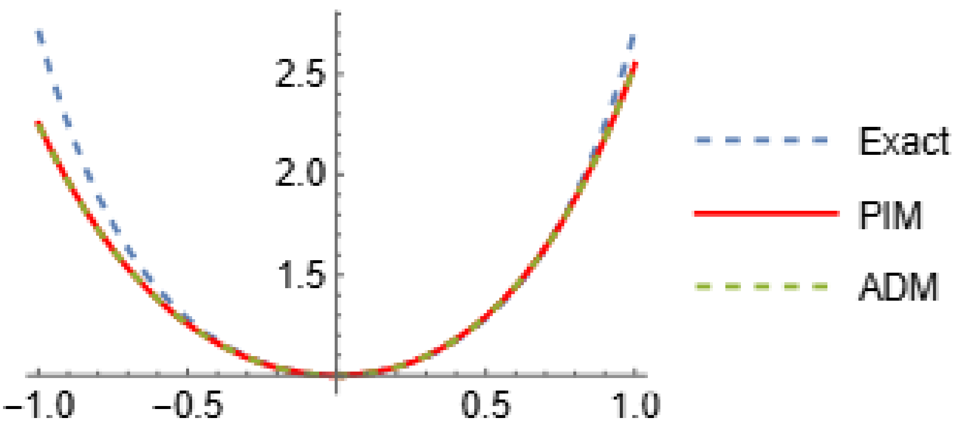

Figure 1: The exact solution $y = e^{x^2}$, PIM and the ADM solutions.

Example 3.2
We study the following second-order nonlinear differential equation:

$$y'' + \frac{1}{x}y' = x^2e^{\frac{x^2}{2}} + 2e^{\frac{x^2}{2}} + e^{x^2} + y^2. \tag{3.6}$$

With initial conditions:

$$y(0) = 1, \quad y^{'}(0) = 0.$$

The exact solution is given by:

$$y(x) = e^{\frac{x^2}{2}}.$$

Solution using PIM

From (2.2) the differential operator becomes such as,

$$L(\cdot) = x^{-1}\frac{d}{dx}x\frac{d}{dx}(\cdot)$$

We can rewrite the equation as:

$$Ly = x^2e^{\frac{x^2}{2}} + 2e^{\frac{x^2}{2}} + e^{x^2} + y^2. \tag{3.7}$$

Where

$$Ly = x^{-1}\frac{d}{dx}x\frac{d}{dx}y,$$

And their inverse operators are given by:

$$L^{-1}(\cdot) = \int_0^x x^{-1}\int_0^x x(\cdot)dxdx.$$

Applying the corresponding inverse operators, we obtain:

$$y = L^{-1}y_n^2. \tag{3.8}$$

Thus, solving for the particular solution:

$$y_0 = 1 + \frac{3x^4}{32} + \frac{7x^6}{288} + \frac{3x^8}{1024} + \frac{11x^{10}}{38400}$$

$$y_1 = 1 + \frac{x^2}{4} + \frac{3x^4}{32} + \frac{17x^6}{576} + \frac{17x^8}{4608} + \frac{133x^{10}}{307200}$$

$$y_2 = 1 + \frac{x^2}{4} + \frac{x^4}{8} + \frac{x^6}{32} + \frac{169x^8}{36864} + \frac{61x^{10}}{102400}$$

$$y_3 = 1 + \frac{x^2}{4} + \frac{x^4}{8} + \frac{19x^6}{576} + \frac{5x^8}{1843200024} + \frac{1273x^{10}}{1843200}$$

$$y_4 = 1 + \frac{x^2}{4} + \frac{x^4}{8} + \frac{19x^6}{576} + \frac{91x^8}{18432} + \frac{13x^{10}}{18432}$$

$$y_5 = 1 + \frac{x^2}{4} + \frac{x^4}{8} + \frac{19x^6}{576} + \frac{91x^8}{18432} + \frac{217x^{10}}{307200}$$

Solution using ADM

Now we will solve (3.6) using the Adomian decomposition method. We can rewrite (3.6) as:

$$Ly = x^2 e^{\frac{x^2}{2}} + 2e^{\frac{x^2}{2}} + e^{x^2} + y^2. \tag{3.9}$$

And

$$L(\cdot) = x^{-1}\frac{d}{dx}x\frac{d}{dx}(\cdot),$$

Applying on the (3.6) and using the initial conditions, we get:

$$y(x) = 1 + \frac{3x^4}{32} + \frac{7x^6}{288} + \frac{3x^8}{1024} + \frac{11x^{10}}{38400} + L^{-1}\left(x^2 e^{\frac{x^2}{2}} + 2e^{\frac{x^2}{2}} + e^{x^2} + y^2\right). \tag{3.10}$$

By assuming that:

$$y(x) = \sum_{n=0}^{\infty} y_n(x).$$

By substituting in the equations, we get:

$$\sum_{n=0}^{\infty} y_n(x) = x^2 + \frac{x^6}{42} + L^{-1}\left(\sum_{n=0}^{\infty} A_n\right).$$

Where

$$y_0 = x^2 + \frac{x^6}{42}, \quad y_{n+1} = L^{-1}\left(\sum_{n=0}^{\infty} A_n\right),$$

Where the Adomian polynomial is given by:

$$A_n = y_n^2.$$

Few components of the Adomian polynomials are:

$$A_0 = y_0^2.$$

$$A_1 = 2y_0 y_1.$$

Hence:

$$y_0 = 1 + \frac{3x^4}{32} + \frac{7x^6}{288} + \frac{3x^8}{1024} + \frac{11x^{10}}{38400}$$

$$y_1 = \frac{x^2}{4} + \frac{x^6}{192} + \frac{7x^8}{9216} + \frac{3x^{10}}{20480}$$

$$y_2 = \frac{x^6}{96} + \frac{x^{10}}{3840}$$

$$y_3 = \frac{x^6}{576} + \frac{x^{10}}{6400}$$

So,

$$y(x) = y_0 + y_1 + y_2 + y_3 = 1 + \frac{x^2}{4} + \frac{3x^4}{32} + \frac{x^6}{24} + \frac{17x^8}{6408} + \frac{87x^{10}}{102400}$$

Table 2: The comparison between exact solution $y(x) = e^{\frac{x^2}{2}}$, PIM and ADM.

x	Exact	PIM	ADM	Exact-PIM	Exact-ADM	PIM-ADM
0.0	1.0	1.0	1.0	0.	0.	0.
0.1	1.00501	1.00251	1.00251	0.00249999	0.0025031	0.00000311633
0.2	1.0202	1.0102	1.01015	0.00999922	0.0100487	0.0000494476
0.3	1.04603	1.02354	1.02329	0.022491	0.0227379	0.000246878
0.4	1.08329	1.04334	1.04257	0.0399486	0.0407139	0.000765247
0.5	1.13315	1.07085	1.06903	0.0623006	0.0641228	0.00182223
0.6	1.19722	1.10783	1.10416	0.0893912	0.0930563	0.00366509
0.7	1.27762	1.1567	1.15015	0.120923	0.127473	0.00654976
0.8	1.37713	1.22075	1.21003	0.156377	0.167095	0.0107184
0.9	1.4993	1.30441	1.28804	0.194888	0.211265	0.0163771
1.0	1.64872	1.41363	1.38996	0.235092	0.258766	0.023674

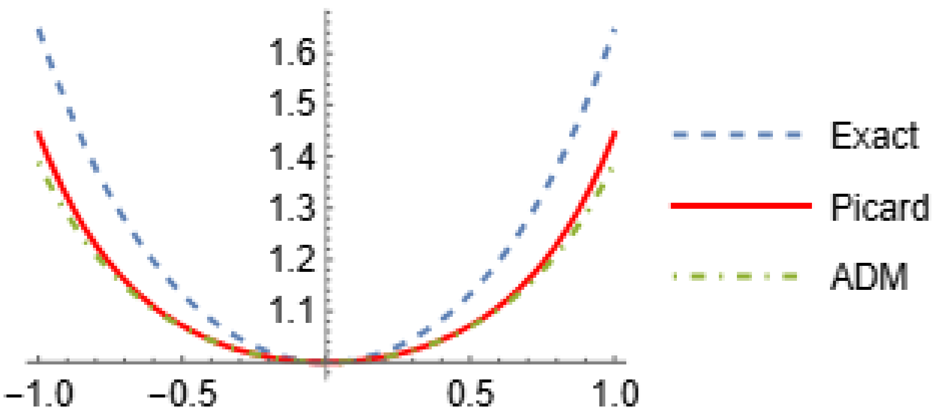

Figure 2: The exact solution $y = e^{\frac{x^2}{2}}$, PIM and the ADM Solution By assuming that $y = \sum_{n=0}^{1} y_n(x)$.

Example 3.3
We consider the nonlinear singular initial value problem [14].

$$y'' + \left(\frac{2}{x}\right) y' = -4(2e^y + e^{\frac{y}{2}}) \tag{3.11}$$

With initial conditions:

$$y(0) = 0, \quad y'(0) = 0.$$

The exact solution is given by:

$$y(x) = -2\ln(1 + x^2).$$

Solution using PIM

From Eq.(3.11) the differential operator becomes such as,

$$L(\cdot) = -4(2e^{y} + e^{\frac{y}{2}})$$

We can rewrite the equation as:

$$Ly = -4(2e^{y} + e^{\frac{y}{2}}). \tag{3.12}$$

Where

$$Ly = x^{-2}\frac{d}{dx}x^{2}\frac{d}{dx},$$

Applying the corresponding inverse operators, we obtain:

$$y = L^{-1}(-4(2e^{y_n} + e^{\frac{y_n}{2}})). \tag{3.13}$$

Thus, solving for the particular solution:

$$y_0 = -2x^2$$

$$y_1 = -2x^2 + \frac{3x^4}{5} - \frac{2x^6}{3} + \frac{x^8}{3}$$

$$y_2 = -2x^2 + x^4 - \frac{4x^6}{7} + \frac{2x^8}{5}$$

$$y_3 = -2x^2 + x^4 - \frac{2x^6}{3} + \frac{92x^8}{189}$$

$$y_4 = -2x^2 + x^4 - \frac{2x^6}{3} + \frac{x^8}{2}$$

Solution using ADM

Now we will solve Eq. (3.11) using the Adomian decomposition method. We can rewrite the system as:

$$Ly = -4(2e^{y} + e^{\frac{y}{2}}) \tag{3.14}$$

And their inverse operators are given by:

$$L(\cdot) = x^{-2}\frac{d}{dx}x^{2}\frac{d}{dx},$$

Applying on the (3.6) and using the initial conditions, we get:

$$y(x) = -2\left(x^2 - \frac{x^4}{2} + \frac{x^6}{3} - \frac{x^8}{4} + \frac{x^{10}}{5} - \frac{x^{12}}{6} + \ldots\right). \tag{3.15}$$

By assuming that:

$$y(x) = \sum_{n=0}^{\infty} y_n(x).$$

By substituting in the equations, we get:

$$\sum_{n=0}^{\infty} y_n(x) = -2\left(x^2 - \frac{x^4}{2} + \frac{x^6}{3} - \frac{x^8}{4} + \frac{x^{10}}{5} - \frac{x^{12}}{6}\right) + L^{-1}\left(\sum_{n=0}^{\infty} A_n\right).$$

Where

$$y_0 = -2x^2, \quad y_{n+1} = L^{-1}\left(\sum_{n=0}^{\infty} A_n\right),$$

Where the Adomian polynomials are given by:

$$A_n = (2e^{y_n} + e^{\frac{y_n}{2}}).$$

Few components of the Adomian polynomials are:

$$A_0 = (2e^{y_0} + e^{\frac{y_0}{2}}).$$
$$A_1 = y_1(2e^{y_0} + \frac{1}{2}e^{\frac{y_0}{2}}).$$

Hence:

$$y_0 = -2x^2$$
$$y_1 = x^4$$
$$y_2 = \frac{2x^6}{3}$$
$$y_3 = \frac{x^8}{2}$$

So,

$$y(x) = -2\left(x^2 - \frac{x^4}{2} + \frac{x^6}{3} - \frac{x^8}{4} + \frac{x^{10}}{5} - \frac{x^{12}}{6}\right)$$

Table 3: The comparison between exact solution $y(x) = -2\ln(1+x^2)$ and PIM, ADM.

x	Exact	PIM	ADM	Exact-PIM	Exact-ADM	PIM-ADM
0.0	0.0	0.0	0.0	0.	0.	0.
0.1	-0.19062	-0.0199007	-0.0199007	0.17072	0.17072	3.96667×10^{-11}
0.2	-0.364643	-0.0784414	-0.0784414	0.286202	0.286202	3.95947×10^{-8}
0.3	-0.524729	-0.172353	-0.172355	0.352375	0.352373	2.18481×10^{-6}
0.4	-0.672944	-0.296803	-0.296839	0.376141	0.376105	0.0000363506
0.5	-0.81093	-0.445964	-0.446273	0.364967	0.364657	0.000309245
0.6	-0.940007	-0.613106	-0.614799	0.325208	0.325208	0.00169305
0.7	-1.06126	-0.789509	-0.796194	0.265063	0.265063	0.00668525
0.8	-1.17557	-0.961277	-0.98132	0.194254	0.194254	0.0200432
0.9	-1.28371	-1.10296	-1.14829	0.135419	0.135419	0.0453282
1.0	-1.38629	-1.16667	-1.23333	0.152961	0.152961	0.0666667

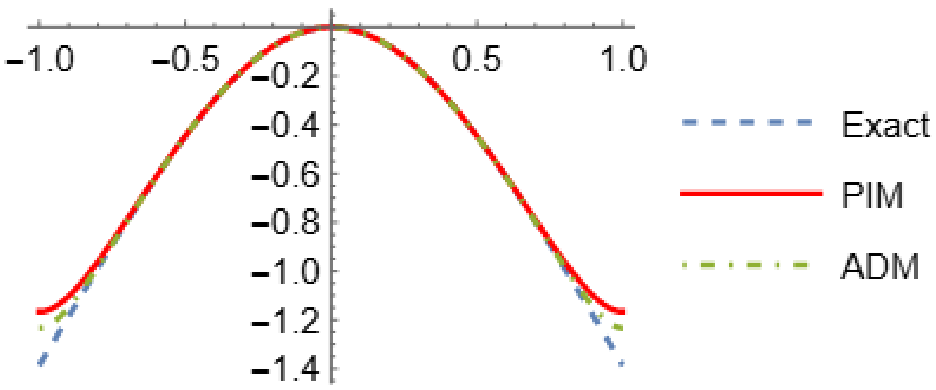

Figure 3: The exact solution $y(x) = -2\ln(1+x^2)$, PIM and ADM.

Example 3.4
In what follows, examples of this type are considered.

$$y^{''} + 2xy^{'} = 6x + 6x^2 - x^6 + y^2, \tag{3.16}$$
$$y(0) = 0, \quad y^{'}(0) = 0,$$

where the exact solution is

$$y(x) = x^3.$$

Solution using PIM

From (2.2) the differential operator becomes such as,

$$L(\cdot) = e^{-x^2}\frac{d}{dx}e^{x^2}\frac{d}{dx}(\cdot), \tag{3.17}$$
$$Ly = 6x + 6x^2 - x^6 + y^2. \tag{3.18}$$

The inverse operator L^{-1} ,as follows,

$$L^{-1}(\cdot) = \int_0^x e^{-x^2}\int_0^x e^{x^2}(y'' + 2xy')dxdx = \int_0^x e^{-x^2}\int_0^x e^{x^2}(6x + 6x^2 - x^6 + y^2)dxdx$$
$$+ \int_0^x e^{-x^2}\int_0^x e^{-x^2}(y^2)dxdx, \tag{3.19}$$

from (3.19) we have that:

$$y(x) = x^3 + \frac{x^4}{2} - \frac{3x^5}{10} - \frac{2x^6}{15} + \frac{x^7}{14} + \frac{3x^8}{280} - \frac{x^9}{72} - \frac{x^{10}}{225} + \int_0^x e^{-x^2}\int_0^x e^{-x^2}(y^2)dxdx. \tag{3.20}$$

So we find

$$y_0(x) = x^3 + \frac{x^4}{2} - \frac{3x^5}{10} - \frac{2x^6}{15} + \frac{x^7}{14} + \frac{3x^8}{280} - \frac{x^9}{72} - \frac{x^{10}}{225},$$

and

$$y_{n+1}(x) = y_0 + L^{-1}y_n^2.$$

Approximations to the solutions are as follows:

$$y_1(x) = x^3 + \frac{x^4}{2} - \frac{3x^5}{10} - \frac{2x^6}{15} + \frac{x^7}{14} + \frac{x^8}{35} - \frac{113x^{10}}{12600}.$$

Solution using ADM

Now we will solve (3.16) using the Adomian decomposition method.

$$L(\cdot) = e^{-x^2}\frac{d}{dx}e^{x^2}\frac{d}{dx}(\cdot),$$
$$Ly = 6x + 6x^2 - x^6 + y^2. \tag{3.21}$$

Where

$$L^{-1}(\cdot) = \int_0^x e^{-x^2}\int_0^x e^{x^2}(y'' + 2xy')dxdx = \int_0^x e^{-x^2}\int_0^x e^{x^2}(6x + 6x^2 - x^6 + y^2)dxdx$$
$$+ \int_0^x e^{-x^2}\int_0^x e^{-x^2}(y^2)dxdx, \tag{3.22}$$

from (3.22) we have that:

$$y(x) = x^3 + \frac{x^4}{2} - \frac{3x^5}{10} - \frac{2x^6}{15} + \frac{x^7}{14} + \frac{3x^8}{280} - \frac{x^9}{72} - \frac{x^{10}}{225} + \int_0^x e^{-x^2}\int_0^x e^{-x^2}(y^2). \tag{3.23}$$

So, we find

$$y_0(x) = x^3 + \frac{x^4}{2} - \frac{3x^5}{10} - \frac{2x^6}{15} + \frac{x^7}{14} + \frac{3x^8}{280} - \frac{x^9}{72} - \frac{x^{10}}{225},$$

and

$$y_{n+1} = L^{-1}y^2 = L^{-1}A_n.$$

Using the Adomian Decomposition Method, we compute the next terms in the series:

$$y_1(x) = \frac{x^8}{56} + \frac{x^9}{72} - \frac{89x^{10}}{12600}.$$

And finally, the general solution:

$$y(x) = y_0 + y_1 + y_2 + \dots$$

So

$$y(x) = x^3 + \frac{x^4}{2} - \frac{3x^5}{10} - \frac{2x^6}{15} + \frac{x^7}{14} + \frac{x^8}{35} - \frac{113x^{10}}{12600},$$

which in a closed form is given by

$$y(x) = x^3.$$

Table 4: The comparison between exact solution $y(x) = x^3$, PIM and the ADM.

x	Exact	PIM	ADM	Exact-PIM	Exact-ADM	PIM-ADM
0.0	0.000	0.0	0.0	0.0	0.0	0.0
0.1	0.001	0.00104687	0.00104687	0.0000468741	0.0000468741	0.0000
0.2	0.008	0.00869645	0.00869645	0.00069453	0.000696453	0.0000
0.3	0.027	0.0302412	0.0302412	0.00324124	0.00324124	0.0000
0.4	0.064	0.0733167	0.0733167	0.00931668	0.00931668	0.0000
0.5	0.125	0.145453	0.145453	0.0204526	0.0204526	0.0000
0.6	0.216	0.253676	0.253676	0.0376764	0.0376764	0.0000
0.7	0.343	0.404219	0.404219	0.0612187	0.0612187	0.0000
0.8	0.512	0.602354	0.602354	0.0903537	0.0903537	0.0000
0.9	0.729	0.85238	0.85238	0.12338	0.12338	0.0000
1.0	1.000	1.1577	1.1577	0.157698	0.157698	0.0000

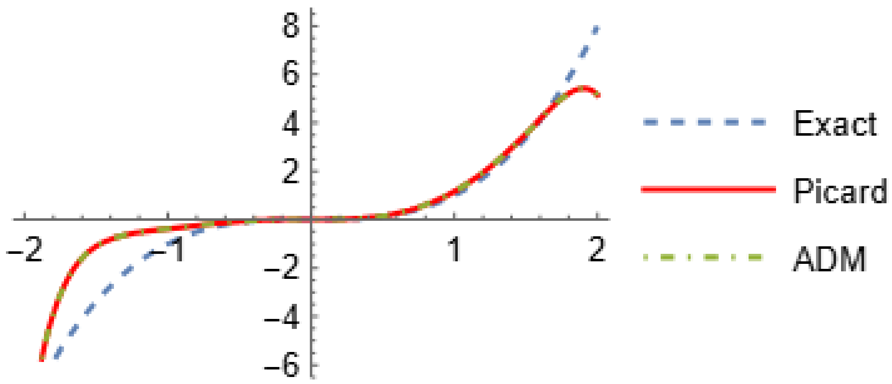

Figure 4: The exact solution $y = x^3$ and the PIM for $y_1, y_2, y_3, \ldots$, ADM solution $y = \sum_{n=0}^{2} y_n(x)$.

4 CONCLUSIONS

This study examines the convergence of the Picard iteration method for second-order nonlinear differential equations in both regular and singular cases. The results indicate that the method provides consistent approximations in regular problems, while its convergence rate may vary in the presence of singular coefficients. A comparison with the Adomian Decomposition Method and exact solutions confirms the reliability of the obtained approximations. Overall, the findings clarify the behavior of Picard-type iterations and supports their use in nonlinear modeling problems involving different coefficient structures.

Author Contributions

Wajeeda Abud AL Rashid contributed to the investigation, computations, and writing of the original draft. Mohammed Ali Qasem Mohsen contributed to conceptualization, methodology, formal analysis, and manuscript revision. Yahya Qaid Hasan contributed to supervision, validation, and critical review. All authors read and approved the final manuscript.

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Conflicts of Interest

The authors declare no conflict of interest.

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